of the amount of current makes the strata distinct and steady. Most frequently a point of steadiness, sufficient for photographing, is produced by the careful introduction of external resistance; subsequently the introduction of more resistance produces a new phase of unsteadiness, and still more resistance another phase of steady and distinct stratification.

7. *The greatest heat is in the vicinity of the strata.* This can be best observed when the tube contains either only one stratum, or a small number separated by a broad interval. There is reason to believe that even in the dark discharge there may be strata; for we have found a development of heat in the middle of a tube, in which there was no illumination except on the terminals.

8. *Even when the strata are to all appearance perfectly steady, a pulsation can be detected in the current; but it is not proved that the strata depend upon intermittence.*

9. *There is no current from a battery through a tube divided by a glass division into two chambers, and the tube can only be illuminated by alternating charges.*

10. *In the same tube and with the same gas, a very great variety of phenomena can be produced by varying the pressure and the current. The luminosities and strata, in their various forms, can be reproduced in the same tube, or in others having similar dimensions.*

11. *At the same pressure and with the same current, the diameter of the tube affects the character and closeness of the stratification.*

II. "Note on Legendre's Coefficients." By I. TODHUNTER, F.R.S. Received April 16, 1878.

In the "Proceedings of the Royal Society," vol. xxvii, pp. 63–71, Professor Adams has given, by an inductive process, the development of the product of any two of Legendre's Coefficients in a series of the Coefficients; from this is immediately deduced the value of the integral between the limits -1 and $+1$ of the product of any three of the Coefficients. On the other hand, if we know the value of this definite integral, we can immediately deduce the development of the product of any two of the Coefficients. Thus it may be of interest to give a brief investigation of the value of the definite integral. I follow the notation adopted by Professor Adams.

The formula to be established is

$$\int_{-1}^{1} \mathrm{P}_m \mathrm{P}_n \mathrm{P}_p d\mu = \frac{2}{2s+1} \frac{\mathrm{A}(s-m)\mathrm{A}(s-n)\mathrm{A}(s-p)}{\mathrm{A}(s)},$$

where $2s = m+n+p$, and the functional symbol $\mathrm{A}(r)$ is thus defined: if r is a positive integer

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$$A(r)=\frac{1\,.\,3\,.\,5\,.\,.\,(2r-1)}{1\,.\,2\,.\,3\,.\,.\qquad r},$$

and in all other cases $A(r)$ is to be considered zero, except when $r=0$, and then it is to be considered $=1$.

It will be observed that $A(r+1)=\frac{2r+1}{r+1}A(r)$, and this is the essential property of the function for our purpose. The demonstration consists of three parts.

I. The proposed formula holds when $m=0$, whatever n and p may be. This is seen to be true in virtue of the known theorem that $\int_{-1}^{1} P_n P_p d\mu$ is zero when n and p are unequal, and is equal to $\frac{2}{2n+1}$ when $p=n$.

II. The proposed formula holds when $m=1$, whatever n and p may be. For by a known theorem we have

$$P_1 P_n=\frac{n+1}{2n+1}P_{n+1}+\frac{n}{2n+1}P_{n-1},$$

therefore

$$\int_{-1}^{1} P_1 P_n P_p d\mu=\int_{-1}^{1}\left(\frac{n+1}{2n+1}P_{n+1}+\frac{n}{2n+1}P_{n-1}\right)P_p d\mu\,;$$

and this vanishes unless $p=n+1$ or $n-1$, and then its value is $\frac{2s}{(2s+1)(2s-1)}$, where $2s=n+p+1$. And if we examine the formula to be established we shall find if $m=1$ the product $A(s-n)A(s-p)$ vanishes, except when $p=n+1$ or $n-1$, and then it becomes unity. Also $\frac{A(s-1)}{A(s)}=\frac{s}{2s-1}$; and thus the required result is obtained.

III. We shall now give an inductive process by which we show that if the formula holds for all values of n and p combined with the values $m-1$ and m of the third integer, it will hold for all values of n and p combined with the value $m+1$ of the third integer.

For $$P_{m+1}P_n P_p=\left\{\frac{2m+1}{m+1}P_m P_1-\frac{m}{m+1}P_{m-1}\right\}P_n P_p$$

$$=\frac{2m+1}{m+1}P_m P_p\left\{\frac{n+1}{2n+1}P_{n+1}+\frac{n}{2n+1}P_{n-1}\right\}-\frac{m}{m+1}P_{m-1}P_n P_p.$$

Thus we can express $\int_{-1}^{1} P_{m+1}P_n P_p d\mu$ by means of cases of the formula already established. If we now put $2s=m+n+p+1$ we thus obtain for the definite integral $\int_{-1}^{1} P_{m+1}P_n P_p d\mu$

$$\frac{2}{2s+1}\,\frac{2m+1}{m+1}\,\frac{n+1}{2n+1}\,\frac{A(s-m)\,A(s-n-1)\,A(s-p)}{A(s)}$$

$$+\frac{2}{2s-1}\,\frac{2m+1}{m+1}\,\frac{n}{2n+1}\,\frac{A(s-1-m)\,A(s-n)\,A(s-1-p)}{A(s-1)}$$

$$-\frac{2}{2s-1}\,\frac{m}{m+1}\,\frac{A(s-m)\,A(s-n-1)\,A(s-1-p)}{A(s-1)}.$$

It is now a matter of easy algebraical work to put these three terms together by the aid of the essential property of the function $A(r)$. If we put the second and third terms together we shall get

$$\frac{2}{(m+1)(2n+1)}\,\frac{(m-n)(s-p)}{(s-m)(2s-2n-1)}\,\frac{A(s-m-1)\,A(s-n)\,A(s-p)}{A(s)},$$

and the first term is equal to

$$\frac{2}{2s+1}\cdot\frac{(2m+1)(n+1)(2s-2m-1)(s-n)}{(m+1)(2n+1)(s-m)(2s-2n-1)}\,\frac{A(s-m-1)A(s-n)A(s-p)}{A(s)};$$

the aggregate of the last two expressions is

$$\frac{2}{2s+1}\,\frac{A(s-m-1)\,A(s-n)\,A(s-p)}{A(s)};$$

as it should be.

I may observe that the definite integral was discussed by Dr. N. C. Schmit in a dissertation published at Brussels in 1858; but no simple result is there obtained.

III. "On the Spectra of Metalloids. Spectrum of Oxygen." By Arthur Schuster, Ph.D., F.R.A.S. Communicated by J. Clerk Maxwell, F.R.S., Professor of Experimental Physics in the University of Cambridge. Received April 25, 1878.

(Abstract.)

The many unexplained phenomena attending the passage of electricity through gases will probably for some time to come occupy the attention of experimental physicists. It is desirable that the subject should be approached from as many different sides as possible. One of our most powerful instruments of research is the spectroscope, but before it can be applied to the study in question we have to settle the chemical origin of the different spectra, which we observe in vacuum tubes, and to discuss in what way such spectra are liable to change under different circumstances. A special investigation has to be made